Division vs DIV - Valentin's findings

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2nd April, 2023, 18:22 **Post: #1**

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Division vs DIV - Valentin's findings

I feel this observation is worth a discussion, from over on the latest Pi Day offering:

> **Valentin Albillo Wrote:** (29th March, 2023 03:16)
>
> Indeed **IP(T/(K*K))** and **T DIV (K*K)**, which would appear at first sight to be equivalent, do really differ at times (though *very* rarely and for large values of **T**, it seems,) when the former's *rounding* does not match the latter's *truncation*.
>
> A trivial program I wrote (relatively) quickly finds all mismatches for various very large integer **T** and for **K** from *2* to *IP(√T)* (i.e. ~ *one million* possible cases for the first eight values of **T** listed):

```
          T              # Mismatches            K
-------------------------------------------------------------
999,999,999,999       31 instances      2, 5, 8, 16, 20, ...
999,999,999,998       19 instances      2, 3, 8, 20, 25, ...
999,999,999,997       12 instances      3, 8, 25, 80, ...
999,999,999,996        3 instances      3, 3125, 31250, ...
999,999,999,995        3 instances      2, 3, 254
999,999,999,994        1 instance       254
999,999,999,993        1 instance       254
999,999,999,992        0 instances       -
 99,999,999,999        0 instances       -
```

> As you can see, for ***T*** = ***999,999,999,999*** there are **31 different instances** (in about a million) where `IP(T/(K*K))` differs from `T DIV (K*K)`, for ***K*** ranging from *1* to *IP(√T)*. The instances begin at ***K*** = **2** (*249,999,999,999* vs. *250,000,000,000*, respectively) and end at ***K*** = **500,000** (*3* vs. *4*, respectively).
>
> Doing the same with ***T*** = ***999,999,999,998***, there's just ***19*** instances reported instead of *31*, and with ***T*** = ***999,999,999,997*** just ***12***. By the time ***T*** equals *999,999,999,995*, a mere ***3*** faulty instances remain (namely for *K* = *2*, *3* and *254*), then *999,999,999,994* and *999,999,999,993* have just the *one* mismatch *(in a million !)* and for *999,999,999,992* and below there seems to be *none*.
>
> Also, as expected, running this small program for input values with less than 12 digits, say ***T*** = **99,999,999,999** instead, i.e. **1E11 - 1**, no instances of mismatches appear at all, and probably the same happens for all smaller ***T***.

I find myself caught in a superposition of states: a lack of surprise that sometimes division will round upwards, and a great surprise that this rounding happens so very rarely in this experiment.

Checking a few of the examples, it seems that division on this 12 digit calculation will round upwards if the 13th digit would be 5 or greater. Is it obvious as to why this should happen so rarely - am I missing something?

Why these particular divisors, and why should divisors come and go as we traverse the table? Is there pattern here which I'm not seeing?

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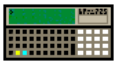
2nd April, 2023, 19:16 **Post: #2**



J-F Garnier
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RE: Division vs DIV - Valentin's findings

These cases are not rare at all, as soon as the dividend is >1E11 such as **(4E11+7)/4 --> 10000000002** that clearly illustrates what is happening.
Although I don't have a rigorous reasoning, I don't believe the problem can occur with dividend X<1E11, so IP(X/Y) is then safe for integer division.
This is not specific to the 71B or Saturn machines, it happens as well on the 10-digit 41C and 34-digit Free42 with the 1E9 and 1E33 limits, respectively.

Rule of the thumb: pay attention to integer divisions when the dividend is less than a factor of 10 from the maximum integer.

J-F

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2nd April, 2023, 19:31 **Post: #3**

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RE: Division vs DIV - Valentin's findings

Mmm, but... if what we had was a rounding from a 13th digit, wouldn't we see it about half of the time? We see it rarely, and I think we don't believe there is a 13th digit, so there's something about the mechanics of division here which is, I think, a bit surprising.

(These are decimal calculations... division proceeds by shift and subtract? Is there an estimating of the next digit?)

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